BEATTY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2010

SUBJECT : Mathematics
PAPER : 4016/01
SETTER : Mrs Samsol

LEVEL : Sec 4E/5N/4N
DURATION : 2 hours
DATE : 13 September 2010

| CLASS : | NAME : | REG NO : |
| :--- | :--- | :--- |

## READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place. For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.

The number of marks is given in brackets [ ] at the end of each question or part question. The total number of marks for this paper is 80 .


This paper consists of $\underline{18}$ printed pages (including this cover page)

Compound Interest

$$
\text { Total amount }=P\left(1+\frac{r}{100}\right)^{n}
$$

## Mensuration

Curved surface area of a cone $=\pi r l$
Surface area of a sphere $=4 \pi r^{2}$

$$
\text { Volume of a cone }=\frac{1}{3} \pi r^{2} h
$$

Volume of a sphere $=\frac{4}{3} \pi r^{3}$

$$
\text { Area of triangle } A B C=\frac{1}{2} a b \sin C
$$

Arc length $=r \theta$, where $\theta$ is in radians

$$
\text { Sector area }=\frac{1}{2} r^{2} \theta \text {, where } \theta \text { is in radians }
$$

## Trigonometry

$$
\begin{aligned}
& \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
& a^{2}=b^{2}+c^{2}-2 b c \cos A
\end{aligned}
$$

Statistics

$$
\begin{aligned}
\text { Mean } & =\frac{\sum f x}{\sum f} \\
\text { Standard Deviation } & =\sqrt{\frac{\sum f x^{2}}{\sum f}-\left(\frac{\sum f x}{\sum f}\right)^{2}}
\end{aligned}
$$

1 If Andy sells his painting for $\$ 20800$, he will make a loss of $12 \%$. How much must he sell the painting if he wants to make a profit of $65 \%$ ?

Answer \$

2

| Country | Population |
| :---: | :---: |
| China | 1.2 billion |
| Japan | 126 million |

(a) Express 126 million in standard form.
(b) Find the ratio of the population of Japan to the population of China.

Answer (a)
(b).

3 (a) Given that $3^{p} \times 16=432$, find $p$.
(b) Simplify $\left(\frac{2}{x}\right)^{-2}$.

Answer
(a) $p=$
(b)

4 (a) Express 24 cm as a percentage of 12.5 m .
(b) If $x$ men takes 5 days to build a wall, find an expression for the number of men needed to build the same wall in $y$ days.

Answer (a)
(b)

5 Hafiz invests $\$ 60000$ in a bank which pays compound interest monthly at a rate of $2.4 \%$ per annum. Calculate the total interest earned at the end of 2 years.

$$
\text { Answer } \$
$$

| $\begin{gathered} \text { For } \\ \text { Examiner's } \\ \text { Use } \end{gathered}$ | $6 y$ is inversely proportional to the square of $x$. It is given that $y=6$ for a certain value of $x$. Find the value of $y$ when this value of $x$ is doubled. | $\begin{gathered} \text { For } \\ \text { Examiner's } \\ \text { Use } \end{gathered}$ |
| :---: | :---: | :---: |

7 The diagram shows a prism whose cross-section is an equilateral triangle. $P Q=Q R=R P=6.5 \mathrm{~cm}$ and the length of the prism is 16 cm . Calculate
(a) the area of triangle $P Q R$,
(b) the volume of the prism


Answer (a)
$\mathrm{cm}^{2}$ [1]
(b)
$\mathrm{cm}^{3}$ [1]


9 (a) Find the interior angle of a regular octagon.
(b) Two of the exterior angles of a $n$-sided polygon are $84^{\circ}$ and $56^{\circ}$, while the remaining exterior angles are each equal to $20^{\circ}$. Find $n$.
Answer (a)
(b) $n=$
[2]

For Examiner's Use

10 (a) The $n$th term of a sequence is given by $3 n^{2}-5$.
Write down the first 4 terms.
(b) The first four terms of another sequence are $5,14,29,50 \ldots \ldots$
(i) Write down the next term.
(ii) By comparing this sequence with your answer to (a), write down the $n$th term.

## Answer

(a)
(b) (i)
(ii)

11 In the diagram, $A B C$ is a right-angled triangle in which $B C=28 \mathrm{~cm}$ and $A C=35 \mathrm{~cm} . B C$ is produced to $D$.

Express as a fraction, the value of
(a) $\sin \angle A C B$,
(b) $\cos \angle A C D$.

Answer (a) $\sin \angle A C B=$ ..... [2]
(b) $\cos \angle A C D=$ ..... [1]

For Examiner's Use

12 The base areas of two geometrically similar vases are in the ratio of $16: 25$.
(a) The curved surface area of the larger vase is $645 \mathrm{~cm}^{2}$. Find the curved surface area of the smaller vase.
(b) If the mass of the smaller vase is 4.8 kg , find the mass of the larger vase.
Answer
(a)
$\mathrm{cm}^{2}$ [1]
(b) kg

13 Solve the simultaneous equations

$$
\begin{aligned}
& 2 x+y=3 \\
& 3 x+2 y-12=0
\end{aligned}
$$

$$
\begin{aligned}
& \text { Answer } \quad x= \\
& y=
\end{aligned}
$$

For

14 A map is drawn to a scale of $1: 500000$.
(a) Find the actual distance, in kilometres, represented by 14.3 cm on the map.
(b) A town covers an area of 246 square kilometres. Find, in square centimetres, the area representing the town on the map.
Answer
(a) $\qquad$ km
(b) $\qquad$ $\mathrm{cm}^{2}$ [2]

15 (a) Express $x^{2}-4 x+9$ in the form $(x-a)^{2}+b$.

$$
\begin{equation*}
\text { Answer (a) } \quad x^{2}-4 x+9= \tag{1}
\end{equation*}
$$

(a) Sketch the graph of $y=x^{2}-4 x+9$.


For
Examiner's Use

16 (a) On the Venn diagram shown in the answer space, shade the set $(A \cup B)^{\prime}$.

(b) $\xi=\{x: x$ is a positive integer and $x<20\}$
$P=\{x: x$ is divisible by 3$\}$
$Q=\{x: x$ is a perfect square $\}$
$R=\{x: x$ is a factor of 24$\}$
Find
(i) $P \cup Q$,
(ii) $\mathrm{n}(P \cap R)$

Answer (b) (i)
(ii) Use

17 The diagram shows the speed-time graph of an object.

(a) Find the speed when $t=8$.
(b) Calculate the distance travelled in the first 26 seconds.
(c) The object comes to rest with a constant retardation of $6.5 \mathrm{~m} / \mathrm{s}^{2}$. Find the value of $k$.

Answer (a) ............................m/s [1]
(b)............................ $m$ [1]
(c) $k=$ s [1]

For
Examiner's Use

18 Given that $\overrightarrow{O A}=\binom{-4}{7}$ and $\overrightarrow{A B}=\binom{15}{-5}$.
(a) Find
(i) $|\overrightarrow{A B}|$,
(ii) $\overrightarrow{O B}$.

Answer
(a)(i) units [1]
(ii).
[1]
(b) Given that $\overrightarrow{A C}=\binom{8}{k}$ and $\overrightarrow{A C}=h \overrightarrow{O A}$, find the value of $h$ and the value of $k$.

## Answer

(b) $h=$
$k=$.

For Examiner's Use

19 In the diagram, the points $A, B, C$ and $D$ lie on the circumference of the circle. The diagonals $A C$ and $B D$ intersect at $E$.

(a) Show that triangle $A B E$ is similar to triangle $D C E$.

Answer (a) $\qquad$
$\qquad$
$\qquad$
(b) Given that $A E=3 \mathrm{~cm}, C E=5 \mathrm{~cm}$ and $A E: D E=3: 2$.

Find the value of
(i) $\frac{\text { area of } \triangle A B E}{\text { area of } \triangle D C E}$,
(ii) $\frac{\text { area of } \triangle A D E}{\text { area of } \triangle A D C}$.

Answer (b)(i)
(ii)
(c) If $\angle B A C=58^{\circ}$ and $\angle B C D=88^{\circ}$, find $\angle D B C$.

## Answer (c)

For Use

20180 can be expressed as a product of its prime factors as $2^{2} \times 3^{2} \times 5$.
(a) Express 480 as a product of its prime factors.
(b) Find the LCM of 480 and 180 .
(c) Find the smallest integer value of $p$ for which $\sqrt{480 p}$ is an integer.
(d) David wants to cover a wall measuring 480 cm by 180 cm with square tiles. Given that only whole tiles are used, find the largest possible length of the side of each tile.

Answer
(a) $480=$
(b) $\mathrm{LCM}=$
(c) $p=$
(d)

21 The coordinates of $A$ and $B$ are $(-2,7)$ and ( $-6,-9$ ) respectively.
(a) Find the length of $A B$.
(b) Find the equation of the line $A B$.
(c) The line $A B$ passes through the point $(2 p, p+1)$. Find the value of $p$.

Answer
(a) $\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$...............................
(b) $\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$ [2]
(c) $p=$

| $\begin{aligned} & \text { For } \\ & \text { Examiner's } \\ & \text { Use } \end{aligned}$ | 22 (a) | Solve the inequality $18-5(2 x-3)>0$. | For Examiner's Use |
| :---: | :---: | :---: | :---: |

Answer (a)
(b) Factorise completely
(i) $m^{3}-m n^{2}$,
(ii) $5 p-10 q-2 p^{2}+4 p q$.

Answer (b)(i)
(ii)
[2]

For Use

23 The waiting times, in minutes, of Red Bus passangers are given in the following table.

## Red Bus

| Time ( minutes ) | $3-5$ | $6-8$ | $10-12$ | $14-16$ |
| :--- | :---: | :---: | :---: | :---: |
| Frequency | 8 | 14 | 9 | 5 |

(a) Calculate
(i) the mean waiting time,
(ii) the standard deviation.
Answer (a)(i) ......................... minutes [1]
(ii) ................................ minutes [2]

The following table shows the waiting time, in minutes, of Blue Bus passengers.

## Blue Bus

Mean waiting time $=8.21$ minutes
Standard deviation $=2.05$ minutes
(b) Which bus passengers would be be more satisfied with the waiting time?

Justify your answer clearly.
Answer (b) : $\qquad$
$\qquad$
(c) Find the probability that a passenger on the Red Bus has to wait $6-8$ minutes.
Answer
(c)
[1]

For
Examiner's Use

24 In the quadrilateral $P Q R S, P Q=8 \mathrm{~cm}, P S=6 \mathrm{~cm}, \angle P Q R=50^{\circ}$ and $R S=5.5 \mathrm{~cm}$.

(a) Complete the quadrilateral $P Q R S$.
(b) On the same diagram, construct the angle bisector of $\angle Q P S$.
(c) Point $T$ is equidistant from $P, Q$ and $S$. By constructing perpendicular bisectors, find and label the point $T$.
(d) Hence write down the length of $T S$.
Answer
(d) $T S=$
.cm [1]

## Answer Key

$1 \quad \$ 39000$
2(a) $1.26 \times 10^{8}$
(b) $21: 200$

3(a) 3
(b) $\frac{x^{2}}{4}$

4(a) $1.92 \%$
(b) $\frac{5 x}{y}$
$5 \quad \$ 2947.22$
$6 \quad 1 \frac{1}{2}$
7(a) $18.3 \mathrm{~cm}^{2}$
(b) $293 \mathrm{~cm}^{3}$

8(a) -13
(b) $x=-\frac{2}{3}$

9(a) $135^{\circ}$
(b) 13

10(a) $-2,7,22,43 \ldots$.
(b) (i) 77
(ii) $3 n^{2}+2$

11(a) $\frac{3}{5}$
(b) $-\frac{4}{5}$

12(a) 412.8 or $412 \frac{4}{5} \mathrm{~cm}^{2}$
(b) 9.375 kg
$13 x=-6, y=15$

14(a) 71.5 km
(b) $9.84 \mathrm{~cm}^{2}$

15(a) $y=(x-2)^{2}+5$
(b)

[2]
16(a)


16(b) (i) $\{1,3,4,6,9,12,15,16,18\}$
(ii) 3

17(a) $14.4 \mathrm{~m} / \mathrm{s}$
(b) 378 m
(c) 28.8 s

18(a)(i) 15.8 units
(ii) $\binom{11}{2}$
(b) -14

19(a) ) $\angle B A E=\angle C D E$ ( angles in the same segment)
$\angle A B E=\angle D C E$ ( angles in the same segment)
$\angle A E B=\angle D E C$ (vert opp angles)
Any two reasons
19(b)(i) $\frac{9}{4}$
(ii) $\frac{3}{8}$
(c) $34^{\circ}$

20(a) $2^{5} \times 3 \times 5$
(b) 1440
(c) 30
(d) 60 cm

21(a) 16.5 units
(b) $y=4 x+15$
(c) -2

22(a) $x<3.3$
(b)(i) $m(m+n)(m-n)$
(ii) $(p-2 q)(5-2 p)$

23(a)(i) $8 \frac{4}{9}$ or 8.44 minutes
(ii) 3.58
(b) Passengers on the Blue Bus are more satisfied as the mean waiting time for the Blue Bus is shorter compared to the Red Bus.
It is also more reliable as the waiting time is more consistent.
24


